Assignment 7.

This homework is due *Tuesday* 11/16/2010.

There are total of 41 points in this assignment. 30 points is considered 100%. If you go over 30 points, you will get over 100% for this homework and it will count towards your course grade.

Collaboration is welcome. If you do collaborate, make sure to write/type your own paper.

This assignment covers sections 5.1, 5.4, 5.6, beginning of 6.1.

(1) REMINDER. Recall definition of a continuous function:
Let $A \subseteq \mathbb{R}$, $f : A \to \mathbb{R}$, $c \in A$. We say that f is continuous at c if

$$\forall \varepsilon > 0\ \exists \delta > 0\ \forall x \in A, \quad \text{if } |x - c| < \delta, \text{ then } |f(x) - f(c)| < \varepsilon.$$

Below you can find (erroneous!) "definitions" of a continuous function. In each case describe, exactly which functions are "continuous at c" according to that "definition".

(a) [4pt] Let $A \subseteq \mathbb{R}$, $f : A \to \mathbb{R}$, $c \in A$. We say that f is "continuous at c" if
$\forall \varepsilon > 0\ \forall \delta > 0\ \forall x \in A, \quad \text{if } |x - c| < \delta, \text{ then } |f(x) - f(c)| < \varepsilon.$

(b) [4pt] Let $A \subseteq \mathbb{R}$, $f : A \to \mathbb{R}$, $c \in A$. We say that f is "continuous at c" if
$\exists \delta > 0\ \forall \varepsilon > 0\ \forall x \in A, \quad \text{if } |x - c| < \delta, \text{ then } |f(x) - f(c)| < \varepsilon.$

(c) [6pt] Let $A \subseteq \mathbb{R}$, $f : A \to \mathbb{R}$, $c \in A$. We say that f is "continuous at c" if
$\forall \varepsilon > 0\ \exists \delta > 0\ \exists x \in A, \quad \text{if } |x - c| < \delta, \text{ then } |f(x) - f(c)| < \varepsilon.$

(2) [3pt] (Exercise 5.4.2) Show that function $f(x) = 1/x^2$ is uniformly continuous on $A = [1, \infty)$, but that it is not uniformly continuous on $B = (0, \infty)$.

(3) (Exercise 5.4.3) Use the Nonuniform Continuity Criterion to show that the following functions are not uniformly continuous on the given sets.

(a) [2pt] $f(x) = x^2$, $A = [0, \infty)$.

(b) [2pt] $g(x) = \sin(1/x)$, $B = (0, \infty)$.

(4) [4pt] (Exercise 5.4.6) Show that if f and g are uniformly continuous on $A \subseteq \mathbb{R}$, and if the are *both* bounded on A, then their product fg is uniformly continuous on A.

(5) [4pt] (Exercise 5.4.8) Show that if f and g are each uniformly continuous on $\mathbb{R}$, then the composite function $f \circ g$ is uniformly continuous on $\mathbb{R}$.

— *see next page* —

(6) (a) [3pt] (Exercise 5.6.4) If f and g are *positive* increasing functions on an interval $I \subseteq \mathbb{R}$, then their product fg is increasing on I.

(b) [2pt] (Exercise 5.6.3) Show that both $f(x) = x$, $g(x) = x - 1$ are strictly increasing on $I = [0, 1]$, but that their product fg is not increasing on I.

(7) (Part of exercise 6.1.1) Use the definition to find derivative of each of the following functions:

(a) [2pt] $f(x) = 1/x$, $x \in \mathbb{R}$, $x \neq 0$

(b) [2pt] $f(x) = \sqrt{x}$, $x > 0$.

(8) [3pt] (Exercise 6.1.2) Show that $f(x) = x^{1/3}$, $x \in \mathbb{R}$, is not differentiable at $x = 0$.